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Hysteresis Phenomena in the Stick-Slip Motion at the Boundary Friction Mode

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The tribological system is considered, which consists of two atomically smooth solid surfaces separated by an ultrathin lubricant film. The thermodynamic model based on the Landau theory of phase transitions is built, which describes behavior of this system in the boundary friction mode. The free energy density for an ultrathin lubricant film is given in the form of expansion into series by the powers of order parameter that is reduced to the shear modulus of lubricant. The kinetics of the system is studied on the basis of model describing first-order phase transitions between kinetic modes of friction. It is shown that in the presence of spring between the external drive and block the width of temperature hysteresis increases versus fixed coupling.

Keywords: Nanotribology, Stick-Slip, Boundary lubrication friction, Non-Newtonian behavior, Viscosity.

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1. INTRODUCTION

In connection with swift nanotechnology development and devices parts miniaturization the problem of guaranteeing stable work of nanomechanism at the deficit or absence of lubricant is more actual. Therefore lately friction laws at the lubricant thickness of several molecular layers are very actively studied. This mode is called boundary friction and it substantially differs from mixed and hydrodynamic friction [1, 2]. In the boundary friction mode the structure of lubricant can realize with domains of liquidlike and solidlike states [3, 4].

As a rule, when the temperature or stress exceeds critical values, the lubricant melts. In the papers [5, 6] a model was built, which describes both situations: usual thermodynamic and "shear" melting. Additionally fluctuations are taken into account [5, 6], because they are important in such microscopical systems, and can be a reason for lubricant transition from solidlike to liquidlike state. Let us note that within the model [5] at the fluctuations presence the stick-slip mode is not strictly periodical, since it has expressed stochastic component. The reasons for hysteresis, which was observed in boundary friction experiments [7, 8], in detail studied within the framework of synergetic model in the paper [9]. Main aim of this research is investigation of hysteresis phenomenon using the thermodynamic model [10] at realization of the first-order phase transition [11].

2. TRIBOLOGICAL SYSTEM AND BASIC EQUATIONS

Let us consider boundary friction using as example the behavior of mechanical analogue of tribological system [3, 4, 12], shown in Fig. 1.

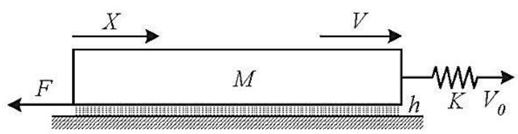

Fig. 1– The scheme of tribological system

The system consists of two blocks with atomically smooth surfaces. The bottom block is fixed, and upper block slides on it. The ultrathin lubricant film with thickness h is between them. The upper block of mass M is connected with a spring with stiffness K, whose free end is inclined to the motion with fixed velocity V_0. When the free end of spring moves the block shears too, but its velocity V differs from velocity V_0, because the friction force F is generated that opposes to motion.

Designating block coordinate as X, we write down the corresponding equation of motion [3, 4, 12, 10]:

$$M\ddot{X} = K\Delta X - F. \tag{2.1}$$

Here ΔX is the spring extension, which can be expressed in the form

$$\Delta X = \int_0^t V_0 \mathrm{d}t' - X, \tag{2.2}$$

where $t = t'$ is the motion time of spring free end.

Friction force F between blocks is calculated using equation [12, 13]

$$F = \left[\sigma_{el} + k\cdot \mathrm{sgn}(V)\left(\frac{|V|}{h}\right)^{\gamma+1}\right]A, \tag{2.3}$$

where σ_{el} is the shear component of elastic stress which arises in the lubricant during motion, A is the friction surface contact area. Also in the equation we introduce a phenomenological coefficient k (Pa·s$^{\gamma+1}$) and dimen-

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sionless exponent γ, which fixes the dependence of effective viscosity of non-Newtonian lubricant on velocity gradient [12, 13, 14, 15]. The case $\gamma < 0$ corresponds to pseudoplastic liquids, value $\gamma > 0$ meets dilatant lubricant, and $\gamma = 0$ describes Newtonian liquid.

Let us write down free energy density for an ultrathin film in the form of expansion in terms of order parameter φ, which represents the amplitude of the periodic part of the microscopic density of the medium [10, 11, 16]:

$$f = \alpha(T - T_c)\varphi^2 + \frac{a}{2}\varphi^2\varepsilon_{el}^2 - \frac{b}{3}\varphi^3 + \frac{c}{4}\varphi^4, \qquad (2.4)$$

where T is the lubricant temperature, T_c is the critical temperature, ε_{el} is the shear component of elastic strain, α, a, b, c are positive constants. The parameter φ takes on nonzero value when lubricant is solidlike. The elastic stress is defined as derivative of potential (2.4) with respect to strain $\sigma_{el} = \partial f/\partial\varepsilon_{el}$:

$$\sigma_{el} = \mu\varepsilon_{el} = a\varphi^2\varepsilon_{el}, \qquad (2.5)$$

where $\mu = a\varphi^2$ is the shear modulus [10]. Thus the shear modulus μ possesses zero value in liquidlike state.

In the previous work [17] it was shown that lubricant melts after exceeding the critical value

$$T_{c0} = T_c - \frac{a}{2\alpha}\left(\frac{\tau_\varepsilon V}{h}\right)^2 + \frac{b^2}{8\alpha c}, \qquad (2.6)$$

and solidifies at the value

$$T_c^0 = T_c - \frac{a}{2\alpha}\left(\frac{\tau_\varepsilon V}{h}\right)^2. \qquad (2.7)$$

In this case the temperature width of hysteresis is defined by expressions (2.6) and (2.7)

$$\Delta T = T_{c0} - T_c^0 = \frac{b^2}{8\alpha c} \qquad (2.8)$$

and depends only on expansion constants (2.4).

In the same way we can assign two critical velocities: if the velocity exceeds the value

$$V_{c0} = \frac{h}{\tau_\varepsilon}\sqrt{\frac{2\alpha(T_c - T)}{a} + \frac{b^2}{4ac}} \qquad (2.9)$$

the lubricants melts, and lubricant solidifies, when V becomes less than the value

$$V_c^0 = \frac{h}{\tau_\varepsilon}\sqrt{\frac{2\alpha(T_c - T)}{a}}. \qquad (2.10)$$

In contrast to the previous case the velocity width of hysteresis $\Delta V = V_{c0} - V_c^0$ increases with temperature growth.

3. HYSTERESIS BEHAVIOR

For the further investigation of system kinetics we write down the Landau-Khalatnikov-type equation [18]:

$$\dot{\varphi} = -\delta\frac{\partial f}{\partial f}, \qquad (3.1)$$

where δ is the kinetic coefficient characterizing the inertia properties.

If the upper block is in motion with constant velocity V the temperature width of hysteresis is defined by expression (2.8). In the case of the system functioning depicted in Fig. 1 at the constant velocity of spring free end V_0 the block velocity V substantially depends on spring rigidity K and block mass M. For example, in the case shown in Fig. 1 the interrupted (stick-slip) motion [3, 4, 12] can be realized which is impossible at $V = \text{const}$.

In Fig. 2 dependencies are shown of maximal values of friction force F, elastic σ_{el} and viscous σ_v stresses at the gradual lubricant temperature T increase.

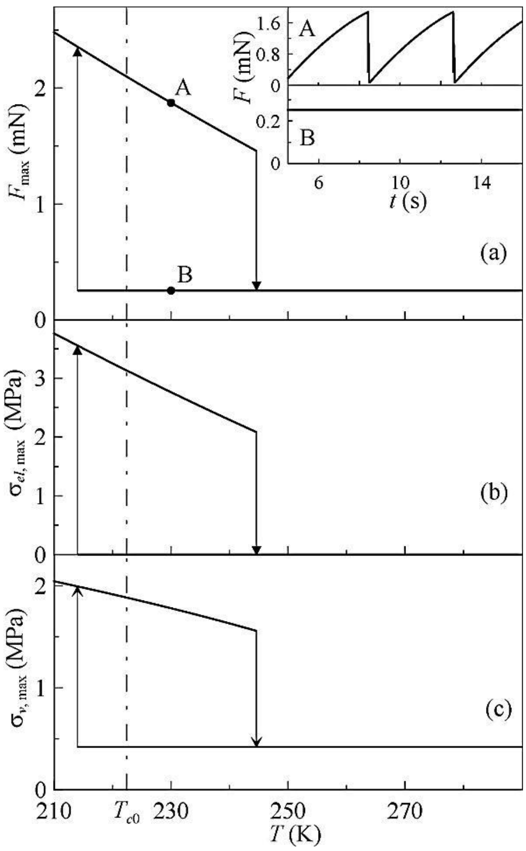

Fig. 2 – The dependencies of maximal values of friction force F (a), elastic σ_{el} (b) and viscous σ_v (c) stresses on lubricant temperature T for the parameters $\alpha = 0.95$ J·K^{-1}/m^3, $T_c = 290$ K, $a = 4\cdot10^{12}$ Pa, $b = 230$ J/m^3, $c = 850$ J/m^3, $h = 10^{-9}$ m, $\tau = 10^{-8}$ s, $\gamma = -2/3$, $A = 0.6\cdot10^{-9}$ m^2, $k = 5\cdot10^4$ Pa·s$^{1/3}$, $\delta = 100$ m^3/s·J^{-1}, $M = 0.4$ kg, $K = 1000$ N/m, $V_0 = 600$ nm/s. In the inset to the panel (a) the time dependencies $F(t)$ are shown corresponding to the points A and B

These dependencies were obtained by numerical solution of the system of kinetic equations (2.1), (3.1). The spring extension ΔX is defined from expression (2.2), friction force F is determined according to (2.3), elastic stress σ_{el} is fixed by expression (2.5). While the system solving, the relationship $\dot{X} = V$ is used. In accordance with the figures at low temperature value T the stick-slip motion is realized, when time dependence of friction force $F(t)$ has saw-like shape (inset A to the figure). In this case periodical phase transitions occur between liquidlike and solidlike lubricant structures. At the temperature increase in the interrupted mode the maximal values of friction force $F_{\max}$, elastic $\sigma_{el,\max}$ and viscous $\sigma_{v,\max}$ stresses decrease. Figure 2 is built at the spring's free end velocity $V_0 = 600$ nm/s. If the block velocity V is fixed, then at the value $V = 600$ nm/s the sliding kinetic regime, which corresponds to the liquidlike lubricant structure, arises at the temperature $T > T_{c0} \approx 222.4$ K (2.6). In Fig. 2 this temperature is shown by vertical dash and dot line. However, since between block and external drive the spring is placed with rigidity K, if the temperature exceeds the value T_{c0} the interrupted friction mode is realized. This regime is characterized by saw-like time dependence of the friction force $F(t)$, as it is shown in the inset in the upper panel of Fig. 2 for the point A at the temperature $T = 230$ K.

Let us consider in more detail the behavior shown in the inset A in Fig. 2. At the beginning of motion the lubricant is solidlike and spring free end starts motion with the velocity $V_0 = 600$ nm/s. Since at the motion the friction force F (2.3) arises the spring stretches out and block velocity V increases slowly. If block velocity exceeds critical value V_{c0}, which according to (2.9) at the temperature $T = 230$ K is approximately 569.1 nm/s, the lubricant melts. And velocity V reaches the maximal value $V_{\max} \approx 45$ μm/s. Block slips on the significant distance, and spring extension ΔX rapidly decreases. The elastic force $K\Delta X$ reduces with the ΔX decreasing, that is the reason for block motion, therefore velocity V decreases too. If the velocity reduces bellow the critical value $V_c^0 \approx 533.9$ nm/s the lubricant solidifies by a scenario of first-order phase transition. The velocity continues to decrease to the minimal value $V_{\min} \approx 4$ nm/s. It should be noted that while the lubricant temperature T increases, the maximal block velocity value $V_{\max}$ decreases and minimal $V_{\min}$ grows. When the temperature increases the peaks frequency rises up on the dependence $F(t)$ [11]. Particularly, at the temperature $T = 242$ K the block velocity reaches the maximal value $V_{\max} \approx 32.69$ μm/s and the minimal $V_{\min} \approx 23$ nm/s. The decreasing in maximal motion velocity explains the decreasing in maximal values of viscous component of the stress σ_v (the second term in the brackets in the equation (2.3)), that is shown by the dashed line in the bottom panel in Fig. 2. It is worth noting that at the initial temperature $T = 210$ K spring extension is so large that after lubricant melting block slips on the significant distance that leads to the spring pressing ($\Delta X < 0$). After that block moves in the opposite direction (the spring becomes straight). At the temperature increasing the same effect is observed approximately to the value $T \approx 224$ K. Note that in Fig. 2 no friction force and stresses amplitudes are shown but their maximal values which are observed in the positive region. Thereby it is revealed that with the lubricant temperature T growth the maximal spring extension ΔX decreases and the minimal block velocity increases. At this at reaching temperature value $T \approx 244.6$ K (the arrow downward in the upper panel in Fig. 2) the minimal block velocity $V_{\min}$ becomes larger than the critical value V_c^0 (2.10) and lubricant does not solidify. With the further temperature increase the lubricant always has liquidlike structure.

If after the complete lubricant melting and the stationary kinetic sliding mode setting in, for which $V = V_0$, the temperature is decreased the lubricant solidifies at smaller temperature T (the arrow upward in the upper panel in Fig. 2), that is significantly fewer than the value of complete melting. In the inset for point B in Fig. 2, which is built at the same temperature with point A, the interrupted mode is absent. The reason for this is that the block velocity V coincides with spring's free end velocity V_0, which is larger than value V_c^0, that is necessary for lubricant solidification. If the lubricant temperature decreases below than the critical value $T_c^0 \approx 214.2$ K the lubricant solidifies. In this case the temperature width of hysteresis is $\Delta T = 30.4$ K.

Now consider the case when the temperature is not increased but the spring's free end velocity V_0 gradually grows (see Fig. 3).

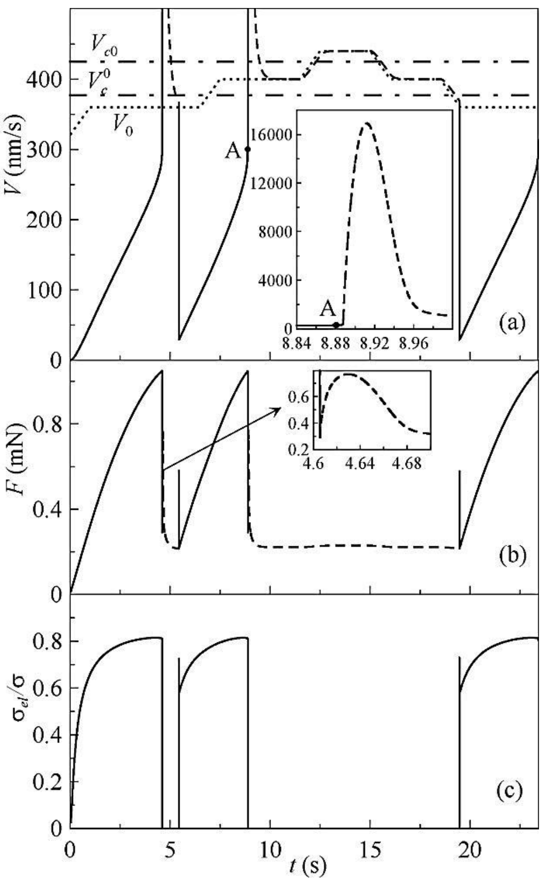

Fig. 3– The time dependencies of block velocity V (a), friction force F (b) and ratio of elastic to total stress σ_{el} / σ (c) for the parameters of Fig. 2 and temperature $T = 260$ K. Spring's free end velocity V_0 in the panel (a) changes and is designated by dotted line, solid line shows block velocity in the solidlike lubricant state, dashed line in the liquidlike state, critical values of melting V_{c0} and solidification V_c^0 are marked by dash and dot lines

In the case of rigid connection with external drive ($V = V_0$) the lubricant melts after exceeding the critical value $V_{c0} \approx 425.9$ nm/s, and solidifies at the velocity

$V_c^0 \approx 377.5$ nm/s. These velocities are shown in the panel (a) in Fig. 3. Let us consider this figure in more detail. At the initial time $t = 0$ the upper block is at rest $V = 0$. At the moment of time $t > 0$ the spring free end is driven with velocity $V_0 = 320$ nm/s and acceleration $a_{ac} = 40$ nm/s^2. During the time period $t = 1$ s velocity V_0 increases to the value 360 nm/s smaller than the critical value V_c^0. At this the upper block velocity monotonically increases, but its value is smaller than V_0 because of spring presence. Then the spring free end moves some time with the constant velocity $V_0 = 360$ nm/s (the horizontal part of dashed dependence), and block velocity V grows. Although velocity V_0 is less than V_{c0}, which is necessary for melting, the lubricant melts with time, because the situation $V > V_{c0}$ is realized due to spring presence. If the velocity V grows the order parameter φ decreases. For the total lubricant melting ($\varphi = 0$) after exceeding the critical velocity V_{c0} some time is necessary, because the system has inertial properties, which are specified by parameter δ in Landau-Khalatnikov equation (3.1). But the liquidlike state is examined where shear modulus is not always equal to zero [3, 4, 12, 13]. Therefore we conditionally consider that lubricant is liquidlike when block velocity exceeds value $V \approx 890$ nm/s because the order parameter $\varphi < 0.01$ and the ratio of elastic stress σ_{el} to the total stress σ is less than 0.7 %. At this the total friction force F abruptly decreases (the panel (b) and the inset to this panel in Fig. 3), and then it begins to increase at the expense of increasing in viscous stress component σ_v. This occurs since block velocity V abruptly increases after melting (see the inset to the upper panel in Fig. 3). At this the spring contracts due to condition $V > V_0$. With the lapse of time the block velocity V decreases to the value less than the critical V_c^0 (2.10) and lubricant solidifies. Thereby the stationary mode of stick-slip motion sets in.

Then during the time of one second spring's free end velocity increases to the value $V_0 = 400$ nm/s (now it is larger than the critical V_c^0, as we can see in figure). According to the figure the lubricant melts again with the velocity rising to the value $V > 16$ μm/s (see inset to the upper panel in Fig. 3). After corresponding extension of spring the block velocity V decreases to the value V_0, but if now $V_0 < V_c^0$ the lubricant does not solidify. With the subsequent spring's free end velocity V_0 increasing the kinetic mode of liquid friction is realized. If now velocity V_0 decreases the lubricant solidifies after fulfillment of condition $V < V_c^0$. If in the liquid friction mode in the stationary state the situation $V = V_0$ realizes at the very slowly velocity V_0 decreasing the lubricant solidifies at $V_0 < V_c^0$. Thus the velocity hysteresis is absent at the chosen parameters, because of spring presence. In this case the velocity can change on several digits. At this if the external drive velocity exceeds the critical value V_c^0 the block velocity becomes larger than V_{c0} with the further lubricant melting. The acceleration value a_{ac} critically effects on the described behavior features. It is expected that at the $a_{ac} \to 0$ at the velocity increasing from zero value the velocity width of hysteresis ΔV is observed.

4. CONCLUSIONS

In the proposed study using Landau theory of the first-order phase transitions the thermodynamic model is built of an ultrathin lubricant film melting confined between two atomically smooth solid surfaces. It is shown that in a wide range of parameters the stick-slip mode is realized when the lubricant periodically melts and solidifies. It is found out that with the temperature or velocity increasing the interrupted mode disappears and the sliding kinetic mode sets in with constant velocity. It is revealed that if the spring is present in tribological system the temperature and velocity hysteresises have different properties. For example, at chosen parameters the velocity hysteresis is possible only at the very slow increase in spring's free end velocity, when the block velocity at solidlike state can relax to the spring's free end velocity. In other cases the velocity hysteresis is not observed. Thus the spring (the elastic properties of system) presence substantially changes the nature of frictional behavior.

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REFERENCES

1. B.N.J. Persson, *Sliding Friction. Physical Principles and Applications* (New York: Springer-Verlag: 2000).
2. A.V. Khomenko, I.A. Lyashenko, *Phys.-Usp.* **55**, 1008 (2012).
3. H. Yoshizawa, J. Israelachvili, *J. Phys. Chem.* **97**, 11300 (1993).
4. A.D. Berman, W.A. Ducker, I.N. Israelachvili, *Langmuir* **12**, 4559 (1996).
5. A.V. Khomenko, Ya.A. Lyashenko, *Tech. Phys.* **55**, 26 (2010).
6. A.V. Khomenko, *Tech. Phys.* **52**, 320 (2007).
7. A.L. Demirel, S. Granick, *J. Chem. Phys.* **109**, 6889 (1998).
8. J. Israelachvili, *Surf. Sci. Rep.* **14**, 109 (1992).
9. A.V. Khomenko, I.A. Lyashenko, *Phys. Sol. State* **49**, 936 (2007).
10. V.L. Popov, *Tech. Phys.* **46**, 605 (2001).
11. I.A. Lyashenko, *Tech. Phys.* **57**, 17 (2012).
12. I.A. Lyashenko, A.V. Khomenko, L.S. Metlov, *Tribol. Int.* **44**, 476 (2011).
13. I.A. Lyashenko, A.V. Khomenko, L.S. Metlov, *J. Frict. Wear* **32**, 113 (2011).
14. I.M. Sivebaek, V.N. Samoilov, B.N.J. Persson, *Phys. Rev. Lett.* **108**, 036102 (2012).
15. G. George Carson, H.-W. Hu, S. Granick, *Tribol. Trans.* **35**, 405 (1992).
16. V.L. Popov, *Sol. State Commun.* **115**, 369 (2000).
17. I.A. Lyashenko, A.M. Zaskoka, *Ukr. J. Phys.* **58**, 91 (2013).
18. L.D. Landau, I.M. Khalatnikov, *Dokl. Akad. Nauk SSSR* **96**, 469 (1954).